NOISE-PARAMETER UNCERTAINTIES FROM MONTE CARLO SIMULATIONS*

J. Randa? and W. Wiatr?

?Radio Frequency Technology Division
National Institute of Standards and Technology
Boulder, CO 80305, U.S.A.

?Warsaw University of Technology
Institute of Electronic Systems
Warsaw, Poland

Abstract: We present results for uncertainties in noise-parameter measurements, obtained using a Monte Carlo simulation of the measurements. Sets of data were generated to simulate measurements on a low-noise amplifier, with given uncertainties in the underlying measurements, including the standard noise temperature (hot or cold), the ambient temperature, the reflection coefficients of the terminations, the scattering parameters of the amplifier, the power measurements, and variations in the connections. Each set of simulated measurement results was analyzed to determine the "measured" noise parameters, and the standard deviation of the set of measured noise-parameter values was computed to determine the uncertainty in each noise parameter. Results are presented for the noise-parameter uncertainties for different values of the underlying measurement uncertainties.

1. Introduction

The problem of uncertainty propagation in measurements of amplifier noise parameters does not admit a simple analytical solution. The four noise parameters are nonlinear functions of the underlying measured quantities, and in a typical measurement they are determined from a least-squares fit to an overdetermined system of equations. We use a Monte Carlo simulation to estimate the uncertainties in "typical" measurements of noise parameters of a low-noise amplifier (LNA) and to investigate how the uncertainties in the noise parameters depend on the uncertainties in the underlying quantities, such as the noise temperature of the non-ambient noise source, the ambient temperature, the reflection coefficients of the terminations, the scattering parameters of the amplifier, the power measurements, and variations in the connections. The following section describes the simulator, as well as the procedures used to compute uncertainties. It also describes the basic set of measurements simulated and analyzed and the terminations used. Section 3 presents the results of the computations, and Section 4 discusses and summarizes the results. A more complete account of this work is contained in [1].

2. Model And Procedures

The standard method for measuring amplifier noise parameters is that suggested by Adamian and Uhlir [2]. A number of different terminations of known reflection coefficient $G_{G,i}$ and noise temperature $T_{G,i}$ are connected to the input of the amplifier, and the output power is measured for each. There is an equation that relates the output power $P_{out,i}$ to the amplifier noise parameters, the amplifier scattering parameters S_{ij}, and the noise temperature and reflection coefficient of the termination. Consequently each measurement yields an equation relating the noise parameters to known or measured quantities. By measuring a number (N_{meas}) of different terminations (usually between 10 and 20), one obtains an overdetermined set of nonlinear equations, which is then solved for the noise parameters by a least-squares fit. The amplifier gain is usually included with the noise parameters in the set of unknowns to be determined, and we do so in this paper.

A good description of the use of Monte Carlo simulation for uncertainty analysis is given in reference [3]. For the simulation, we first chose "true" values for the underlying quantities. These comprise the noise and scattering parameters of the amplifier and the noise temperature and reflection coefficient of each termination. We then chose uncertainties for the S_{ij}, $T_{G,i}$, $G_{G,i}$, and $P_{out,i}$. In this paper, all measurement distributions are taken to be Gaussian. We also chose a value for the connector variability.

We generated simulated measured values for the S_{ij}, $T_{G,i}$, and $G_{G,i}$, in the standard manner, randomly choosing a value from a Gaussian distribution centered at the true value. For the complex quantities, real and imaginary parts were generated independently. To generate the simulated power measurement, we first calculated the true output power from the equation for output power, using the true values for the noise parameters and the termination noise temperatures, and using the true values for the S-parameters and the reflection coefficient *for that connection*. Once the true output power for the given connection was calculated, the measured value was generated using the uncertainty in the power measurement as the standard deviation. A complete simulated measurement set then consisted of the measured values for S_{ij} and the measured $T_{G,i}$, $G_{G,i}$, and $P_{out,i}$ for each of the N_{meas} terminations.

The complete simulated measurement set can be analyzed and the noise parameters and gain determined in the same way as for a real data. The analysis program we used was the eight-term linear model of reference [4,5]. None of the results of the present paper should be sensitive to the particular analysis program used. To assess the uncertainties in the noise parameters, we generated a large number N_{sim} of simulated measurement sets with the given uncertainties in the underlying quantities. Each simulated measurement set was analyzed to produce a set of “measured” noise parameters, yielding N_{sim} measured values for each parameter. The average and standard deviation of the measured values were computed, and the standard deviation was identified as the uncertainty in a single measurement of that quantity. (Statistics for G_{opt} were computed on real and imaginary parts, not on magnitude and phase.)

We tried several different values of N_{sim} , ranging up to 1000. The results for 100 simulations were essentially the same as for 1000, and so we used N_{sim} = 100 in all the simulations presented below. For the complete set of measurements we used 13 different terminations. One was a hot source with $T_{G,1}$ = 9920 K and $G_{G,1}$ = 0.0181 – 0.1215 j . The others were all at ambient temperature ($T_{G,i}$ = 296 K), and their reflection coefficients were distributed around the complex plane as shown in Fig. 1. We have also shown the true value used for G_{opt} in the simulations. In addition to the hot source, there was one matched load, and the other terminations were reflective or partially reflective loads with various phases. We did not investigate in any detail the effect of changing the number of different terminations used. We did test the effect of eliminating one termination [1], but the focus of this paper is the dependence of the noise-parameter uncertainties on the underlying measurement uncertainties.

3. Simulation Results

There are several parameterizations for the noise characteristics of amplifiers. In this paper we consider only a variant of the IEEE set of parameters [6], corresponding to the parameterization of the noise figure given by

$$T_e = T_{e,\min} + t\frac{\left|\Gamma_{opt} - \Gamma_G\right|^2}{\left|1+\Gamma_{opt}\right|^2 (1-\left|\Gamma_G\right|^2)}$$

The true values for the amplifier were chosen to be G = 2399 (33.80 dB), $T_{e,min}$ = 109.6 K (F_{min} = 1.392 dB), G_{opt} = 0.050 + 0.142 j, and t = 176.3 K. These choices correspond to the values measured for a particular LNA at 11 GHz. We denote the standard deviations (or the standard uncertainties [7]) by s_G for the real or imaginary part of the reflection coefficients and for any S-parameter except S_{21} , s_{S21} for the real or

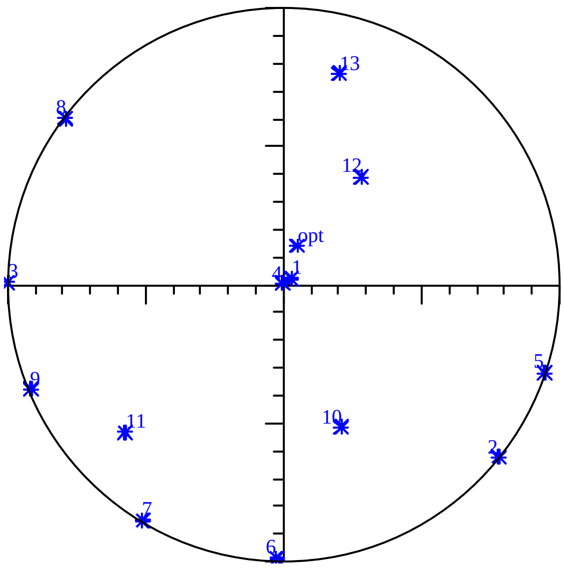

Fig. 1. Distribution of reflection coefficients of terminations in and on unit circle.

imaginary part of S_{21} , and s_{con} for the connector variability. For the ambient noise temperature, the hot noise temperature, and the power measurement, we use *fractional* standard deviations $s_{a,frac}$, $s_{h,frac}$, and $s_{p,frac}$, respectively. The uncertainties estimated in this paper are type-B uncertainties only [7]. In actual applications there will also be type-A uncertainties that are evaluated statistically in the fitting procedure.

A set of values for the underlying uncertainties was chosen to serve as a baseline, to which other results could be compared. For this purpose we used s_{S21} = 0.01, s_{con} = 0.001, $s_{a,frac}$ = 0.005, $s_{h,frac}$ =0.005, and $s_{p,frac}$ = 0.001 For the reflection coefficients and the S-parameters other than S_{21}, we used s_G = 0.002 if the reflection coefficient was 0.5 or less and s_G = 0.003 if the reflection coefficient was greater than 0.5. These choices are not meant to have any particular significance; they just provide a point of comparison. For the most part they are quite good, but achievable, uncertainties. With these underlying uncertainties, the 100 simulated measurement sets yielded the following results: G = 2400 ± 13 (33.80 dB ± 0.02 dB), $T_{e,min}$ = 109.5 K ± 2.5 K, F_{min} = 1.391 dB ± 0.027 dB, t = 176.2 K ± 1.9 K, G_{opt} = (0.050 + 0.140j) ± (0.013 + 0.010j). For G and F_{min} the statistics were done on the linear quantity, and the results were converted to dB.

The effect of increasing the uncertainties in the underlying quantities was investigated by increasing one uncertainty while holding the others fixed at their base values. The results for the power, reflection coefficient, and hot noise temperature uncertainties are given in Tables 1 through 3. In each table, the first row of results is for the baseline set of uncertainties, and the following rows demonstrate the effect of relaxing the pertinent uncertainty. The two entries for s_G in the first row of Table 2 reflect the fact that different uncertainties were used for small |G| and for large |G|, as discussed above. For the other rows of Table 2, the same uncertainty was used for all values of |G|. In Table 3 the final two rows correspond to uncertainties of 0.10 dB and 0.15 dB hot noise temperature. The general features of the results in Tables 1 to 3 are consistent with intuition: the uncertainty in the power measurement affects everything, though not as strongly as one might expect.; s_G has a strong effect on the uncertainty in G_{opt} and weaker effects on G, T_{min}, and t; and the uncertainty in the hot noise temperature has a strong effect on all parameters except G_{opt} , on which it has no effect. A warning is that the present simulation does not include any correlations between successive measurements. Therefore, over the course of the 13 different measurements in each measurement set, the errors tend to cancel or average out. If correlations are present, *e.g.*, if the power readings are systematically high, then the uncertainty could be significantly larger. The effect of correlations on the uncertainties will be studied in subsequent work.

For guidance in estimating achievable uncertainties, we have also computed the uncertainties in noise parameters resulting from a few sets of underlying uncertainties that we consider typical or representative of common situations. The four cases are labeled Average (meant to represent average industrial laboratory measurements), Good (representing measurements at a very good industrial laboratory or a good standards laboratory), and two different very good cases (meant to represent national standards laboratories). Two different very good cases were included in order to test the difference between using two different types of nonambient noise sources. VG-h uses a hot diode source with T = 9200 K and with a noise-temperature uncertainty typical of a good national laboratory calibration, u_T = 0 .5 %. VG-c uses a cryogenic noise source with T = 78 K and with an uncertainty equal to that of NIST's cryogenic primary standard, u_T = 0 .8 %. The underlying uncertainties for these four cases are given in Table 4. The Good entries are the same as the baseline case defined above. Average has a much larger uncertainty in the hot noise source, as might be the case if it were not calibrated by a good standards laboratory, and also has larger uncertainties in the power and reflection coefficient measurements. VG-h is quite similar to Good, but it improves the control of the ambient temperature, and it also has a smaller uncertainty for the large reflection coefficients. VG-c is the same as VG-h except for the nonambient noise temperature and its uncertainty.

$s_{p,frac}$	u_G (dB)	u_{Tmin} (K)	u_{Fmin} (dB)	u_t (K)	u_{ReG}	u_{ImG}
0.001	0.024	2.5	0.027	1.9	0.013	0.011
0.005	0.033	3.5	0.038	2.5	0.020	0.013
0.010	0.051	5.6	0.061	3.6	0.032	0.020

Table 1. Effect of Fractional Power Uncertainty

s_G	u_G (dB)	u_{Tmin} (K)	u_{Fmin} (dB)	u_t (K)	u_{ReG}	u_{ImG}
.002, .003	0.024	2.5	0.027	1.9	0.013	0.011
0.005	0.024	2.7	0.030	2.7	0.020	0.016
0.010	0.027	3.5	0.038	5.0	0.037	0.032

Table 2. Effect of Uncertainty in Reflection Coefficients.

$s_{h,frac}$	u_G (dB)	u_{Tmin} (K)	u_{Fmin} (dB)	u_t (K)	u_{ReG}	u_{ImG}
0.005	0.024	2.5	0.027	1.9	0.013	0.011
0.010	0.048	4.6	0.050	2.6	0.013	0.011
0.020	0.096	8.9	0.098	4.2	0.013	0.011
0.0223	0.112	10.4	0.114	4.8	0.013	0.011
0.0351	1.170	15.6	0.173	7.0	0.013	0.011

Table 3. Effect of Uncertainty in Noise Temperature of Hot Source.

Case	$u_{a,frac}$	$u_{h/c,frac}$	$u_{p,frac}$	u_G	u_{con}	u_{S21}
Average	0.005	0.020	0.002	0.005	0.001	0.010
Good	0.005	0.005	0.001	0.002, 0.003	0.001	0.010
V. Goodh	0.001	0.005	0.001	0.002	0.001	0.010
V. Goodc	0.001	0.008	0.001	0.002	0.001	0.010

Table 4. Underlying Uncertainties Used in Representative Cases

Case	u_G (dB)	u_{Tmin} (K)	u_{Fmin} (dB)	u_t (K)	u_{ReG}	u_{ImG}
Average	0.101	9.0	.099	4.6	.020	.016
Good	0.024	2.5	.027	1.9	.013	.011
V. Goodh	0.024	2.4	.026	1.5	.008	.007
V. Goodc	0.019	1.5	.016	1.4	.008	.007

Table 5. Noise Parameter Uncertainties for Representative Cases

The uncertainties in the noise parameters for these cases are tabulated in Table 5. Two features warrant comment. The seemingly innocuous change in u_G between Good and VG-h has a significant effect on the uncertainty in G_{opt}. Also, VG-c has significantly smaller uncertainties for G and T_{min} (and therefore also for F_{min}) than VG-h, despite having a larger fractional uncertainty in the noise temperature of the source. This is due to the fact that the important temperature uncertainty is in the scale, $T_{amb} - T_{cry}$, which is smaller due to the small uncertainty in T_{amb}.

4. Summary

We performed a Monte Carlo study of the dependence of the uncertainties in measured noise parameters on the uncertainties in the underlying quantities, including hot noise temperature, reflection coefficients of terminations, and power measurements. This was done for a common method of noise-parameter measurement and for a single set of values of noise parameters. We also presented results for the uncertainties for several special cases meant to represent common measurement environments. We have deferred for future work several obvious extensions and generalizations, such as including correlations between underlying measurement uncertainties, allowing nongaussian distributions in generating the simulated measurements, and considering other choices for the noise parameters.

5. References

[1] Randa J. and Wiatr W. Submitted to IEE Proceedings ? Science, Measurement and Technology.

[2] Adamian V. and Uhlir A. IEEE Trans. on Instrum. and Meas., v.IM-22, no.2, pp. 181–182 (1973).

[3] Press W.H., Flannery B.P., Teukolsky S.A., and Vetterling W.T. Numerical Recipes, Ch. 14.5, Cambridge University Press, Cambridge, UK, 1986, 818 p.

[4] Wiatr, W. Proc. of the 10th Microwave Conf. MIKON 94, pp. 525–529, Ksiaz, Poland (1994).

[5] Wiatr W. IEEE Trans. Instrum. and Meas., v.44, no.2, pp. 343–346 (1995).

[6] Haus H. *et al*. Proc. IRE, vol. 48, pp. 60–68 (1960).

[7] *ISO Guide to the Expression of Uncertainty in Measurement*. Geneva, Switzerland (1993).

Digest of the British Electromagnetic Measurement Conference (on CD)

6 – 8 November, 2001
Harrogate, U.K.

Session on Guided Wave and Network Measurements